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RESIDUAL VIBRATION OF A SERVOMOTOR DRIVEN FLEXIBLE BEAM

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In this paper, residual vibration of a servomotor driven rotating flexible beam is studied. The beam is modeled as an Euler-Bernoulli beam; it is rotated by a servomotor using triangular velocity profile (bang-bang trajectory). Analytical solution is also obtained by using Fourier series expansion of the acceleration of the rotating beam. Residual vibration amplitudes depend on the beam tip position at the end of the rotation which is the function of rise time which is the time to complete the rotation. It is found that if the rise time is the odd multiple of the beam period 1, 3, 5 ..., residual vibration amplitudes are maximized. Residual vibration amplitude spectrum show that for rise time to period ratios from 1.5 to 2.5, residual vibration amplitudes are lowered to less than 3% of the maximum residual vibration amplitude obtained for rise time equal to the first natural period of the beam.

1. Introduction

Demand for high performance robotic systems quantified by high speed operation, high end-position accuracy and lower energy consumption has triggered a vigorous research in various multidisciplinary areas, such as design and control of lightweight flexible robot arm. Flexible manipulators, although having some inherent advantageous over conventional rigid robots, have posed more stringent requirements on the control system design, such as accurate end point sensing and fast suppression of transient vibration during rapid arm movements.

Point to point position control of a flexible beam is studied analytically and experimentally[1,2]. Equations for a rotating Timoshenko beam are developed for pinned-free and clamped-free boundary conditions[3]. Dynamic modelling and optimal control of a rotating Euler-Bernoulli beam is studied[4]. Main objective of the paper was to control the vibration through force feedback control. Condition of a slewing beam using high speed camera system is studied[5]. Results show that the natural frequency of the rotating beam is between the natural frequencies of fixed-free and free-free beam. Among many[6-13] are also worth to mention which are related to a rotating flexible beam. A residual vibration spectrum for a rotating flexible beam is studied[13]. In this study cycloidal rise function is used to rotate the beam. Closed loop solution is obtained. Results show that, for frequency ratios of 2, 3, 4 ... residual vibration amplitudes becomes zero.

Shina and Brennan[14] considered two methods for controlling the residual vibrations of a translating or rotating Euler–Bernoulli cantilever beam. Although a beam has an infinite number of vibration modes, when it simply changes its position by translation or rotation the first mode is the main contributor to the total response. Thus, the problem can be reduced to the base acceleration excitation of a single-degree-of-freedom system. Two simple methods are suggested for suppressing the residual vibration of such a system without considering any control algorithms

In this study, servomotor driven flexible beam is considered. Triangular velocity profile (bang-bang trajectory) is used to rotate the beam. Angular acceleration of the beam is approximated by Fourier series and analytic solution is obtained. Residual vibration amplitudes depend on the ratio of rise time to the beam vibration period. For ratios of 1, 3, 5, … residual vibration amplitudes are maximized. It is possible to minimize residual vibration by choosing appropriate ratio of rise time to beam vibration period.

2. Formulation

2.1 Equation of a Rotating Beam

Fig. 1 shows a rotating flexible beam. OXY is an inertial frame, Oxy is a rotating frame. θ is a rotation angle, y is the beam deflection. m is the unit mass of the beam per length.

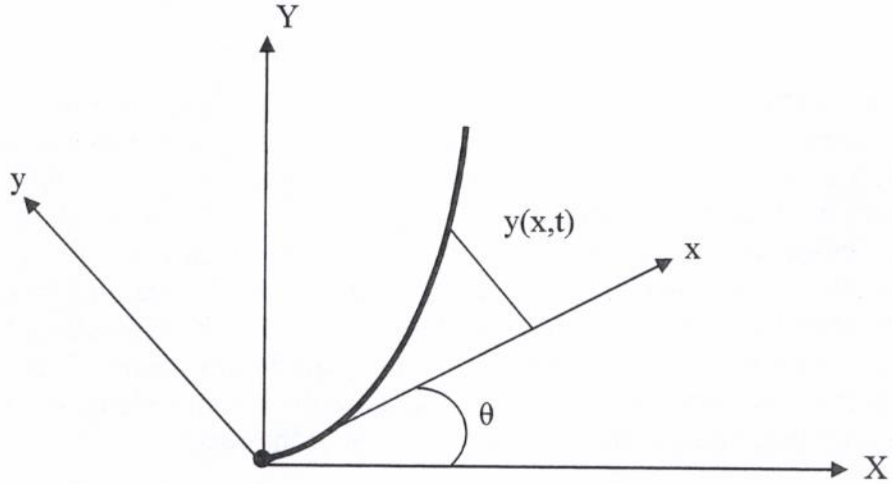

Figure 1. Rotating flexible beam model.

The position vector of m with respect to the rotating coordinates is

$$\vec{r} = x\vec{i} + y\vec{j} . \tag{1}$$

If $\vec{r}$ is derived twice with respect to time

$$\ddot{\vec{r}} = (\ddot{x} - 2\dot{y}\dot{\theta} - y\ddot{\theta} - x\dot{\theta}^2)\vec{i} + (\ddot{y} + 2\dot{x}\dot{\theta} + x\ddot{\theta} - y\dot{\theta}^2)\vec{j} . \tag{2}$$

Acceleration of mass m in the $\vec{j}$ direction will be

$$\ddot{y} + 2\dot{x}\dot{\theta} + x\ddot{\theta} - y\dot{\theta}^2 . \tag{3}$$

Longitudinal vibration is ignored then $\dot{x} = 0$, also nonlinear term $y\dot{\theta}^2$ is neglected. Inertial load on the unit mass m of the flexible beam will be

$$p(x) = -m(\ddot{y} + x\ddot{\theta}) . \tag{4}$$

When this inertial load is used and EI is assumed constant then Euler-Bernoulli equation of the beam will be

$$EI\frac{d^4 y}{dx^4} = -m(\ddot{y} + x\ddot{\theta}) . \tag{5}$$

The governing equation of the motion will be

$$EIy^{iv} + m\ddot{y} = -mx\ddot{\theta}\,. \tag{6}$$

The mode summation method is assumed for the solution which is

$$y(x,t) = \sum_{i=1}^{n} \phi_i(x) q_i(t)\,. \tag{7}$$

If the orthogonality condition is used and viscous damping is assumed, equation for the generalized coordinate q_i is

$$\ddot{q}_i + 2\zeta\omega_i \dot{q}_i + \omega_i^2 q_i = \frac{\Gamma_i}{M_i}\ddot{\theta}(t)\,. \tag{8}$$

Here Γ_i is defined as mode participation factor which is

$$\Gamma_i = -m\int_0^l x\phi_i(x)dx\,. \tag{9}$$

M_i is defined as generalized mass which is

$$M_i = m\int_0^l \phi_i^2(x)dx\,. \tag{10}$$

State space form of the differential equation (8) can be given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta\omega_i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \Gamma_i / M_i \end{bmatrix} \ddot{\theta}(t)\,. \tag{11}$$

2.2 Servomotor Motion

Flexible beam is rotated by a servomotor. Velocity profile is assumed as triangular which is also called bang-bang trajectory. Triangular velocity profile can be given as

$$\dot{\theta}(t) = \frac{2\dot{\theta}_{\max}}{t_r} t \qquad 0 \le t \le \frac{t_r}{2}\,. \tag{12}$$

$$\dot{\theta}(t) = 2\dot{\theta}_{\max} - \frac{2\dot{\theta}_{\max}}{t_r} t \qquad \frac{t_r}{2} \le t \le t_r\,. \tag{13}$$

Here $\dot{\theta}_{\max}$ is the maximum angular velocity, t_r is the rise time. If rise time t_r and rotation angle θ is given, angular velocity and angular acceleration of the rotation can be calculated as

$$\dot{\theta}_{\max} = \frac{2\theta_{\max}}{t_r}\,. \tag{14}$$

$$\ddot{\theta}_{\max} = \frac{2\dot{\theta}_{\max}}{t_r} \quad 0 \le t \le \frac{t_r}{2}\,. \tag{15}$$

$$\ddot{\theta}_{\max} = -\frac{2\dot{\theta}_{\max}}{t_r} \quad \frac{t_r}{2} \le t \le t_r\,. \tag{16}$$

2.3 Analytic Solution

Angular acceleration of the beam rotation for triangular velocity profile is a rectangular wave which can be approximated by a Fourier series as

$$\ddot{\theta}(t) = \ddot{\theta}_{\max} \frac{4}{\pi}\left[\sin \omega_r t + \frac{1}{3}\sin 3\omega_r t + \frac{1}{5}\sin 5\omega_r t + \cdots \right]. \tag{17}$$

Only first three terms are used. $\omega_r = 2\pi / t_r$ is the fundamental frequency of the Fourier series which is also the rise frequency of the beam. Equation (8) can be written again as

$$\ddot{q}_i + 2\zeta\omega_i\dot{q}_i + \omega_i^2 q_i = \frac{\Gamma_i}{M_i}\left(\ddot{\theta}_{\max}\frac{4}{\pi}\right)\left[\sin \omega_r t + \frac{1}{3}\sin 3\omega_r t + \frac{1}{5}\sin 5\omega_r t + \cdots \right]. \tag{18}$$

Solution of the Eq. (18) is

$$q_i = X_0 e^{-\zeta\omega_n t}\sin(\omega_{di} t + \phi_0) + \sum_{n=1,3,5} A_n \sin(\omega_n t - \phi_n). \tag{19}$$

$\omega_{di} = \omega_i\sqrt{1-\zeta^2}$ is the damped natural frequency of the beam and $\omega_n = n\omega_r$. Using initial conditions of $q_i(0) = 0$ and $\dot{q}_i(0) = 0$, X_0 and ϕ_0 will be obtained as

$$\phi_0 = \tan^{-1}\frac{\sum_{n=1,3,5} A_n \sin\phi_n}{\frac{\zeta\omega_i}{\omega_{di}}\sum_{n=1,3,5} A_n \sin\phi_n - \frac{1}{\omega_{di}}\sum_{n=1,3,5} A_n\omega_n \cos\phi_n}. \tag{20}$$

$$X_0 = \sqrt{\left(\sum_{n=1,3,5} A_n \sin\phi_n\right)^2 + \left(\frac{\zeta\omega_i}{\omega_{di}}\sum_{n=1,3,5} A_n \sin\phi_n - \frac{1}{\omega_{di}}\sum_{n=1}^{3} A_n\omega_n \cos\phi_n\right)^2}. \tag{21}$$

Here

$$A_n = \frac{\frac{A_i m}{n\omega_i^2}}{\sqrt{(m-n)^2 + (2\zeta mn)^2}}. \tag{22}$$

$$\phi_n = \tan^{-1}\frac{2\zeta mn}{(m-n)^2}. \tag{23}$$

$$t_r = mT = m\frac{2\pi}{\omega_i}. \tag{24}$$

$$A_i = -\frac{\Gamma_i}{M_i}\frac{4}{\pi}\ddot{\theta}_{\max}. \tag{25}$$

3. Simulation

For the simulation steel beam is used. Properties of the steel beam are; elasticity modulus E=207 GPa, mass density ρ =7700 kg/m^3, length l=40 cm, width b=24 mm, thickness h=0.6 mm. Fig. 2 and 3 show the numerical solution of Eq. (10) for rise time $t_r = 2.2T$ and $t_r = 3T$, respectively. The period of the beam for the first natural frequency is $T = 0.32$ s ($\omega_n = 19.73$ Hz). Damping ratio is assumed as $\zeta = 0.02$. During the rotation, beam is moving under inertial load, when the rotation stops the deflection of the beam at this moment become an initial displacement for the beam's residual vibration. As can be seen from the Fig. 2, for $t_r = 2.2T = 0.70$ s, beam tip displacement is very small that is why residual vibration amplitudes of the beam are small. In Fig. 3, rise time is 3T which is 0.96 s. Beam deflection at 0.96 s is bigger than the one for 2.2T that is why residual vibration amplitudes are high.

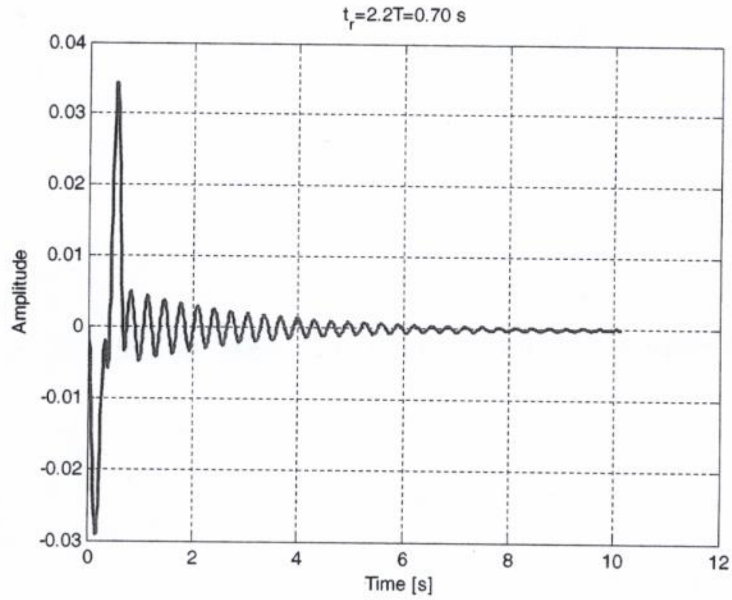

Figure 2. Rotating beam vibration for t_r=2.2T.

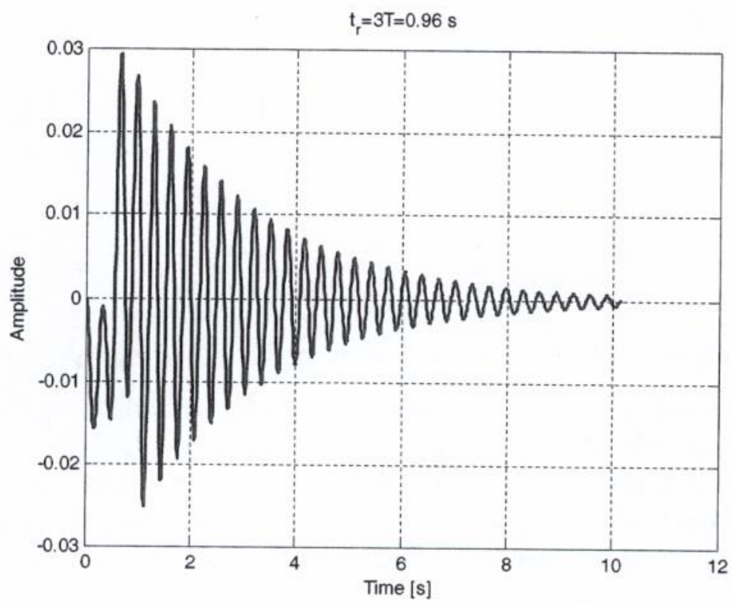

Figure 3. Rotating beam vibration for t_r=3T.

Fig. 4 and 5 show beam vibration during the rotation of the beam for $t_r = 2.2T$ and $t_r = 3T$, respectively. Solid line is for analytical solution, dashed line is for numerical solution which is given in Eq. (18). Analytic solution, which uses three term Fourier series expansion of the acceleration of the beam predicts well the motion of the beam. Depending on the rise time, beam tip amplitude at $t = t_r$ is changing.

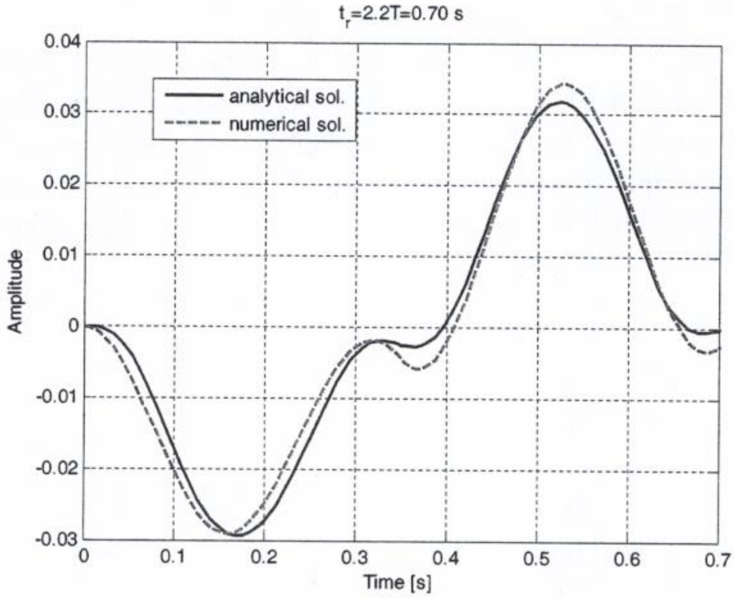

Figure 4. Rotational motion of the beam during rise time, t_r=2.2T.

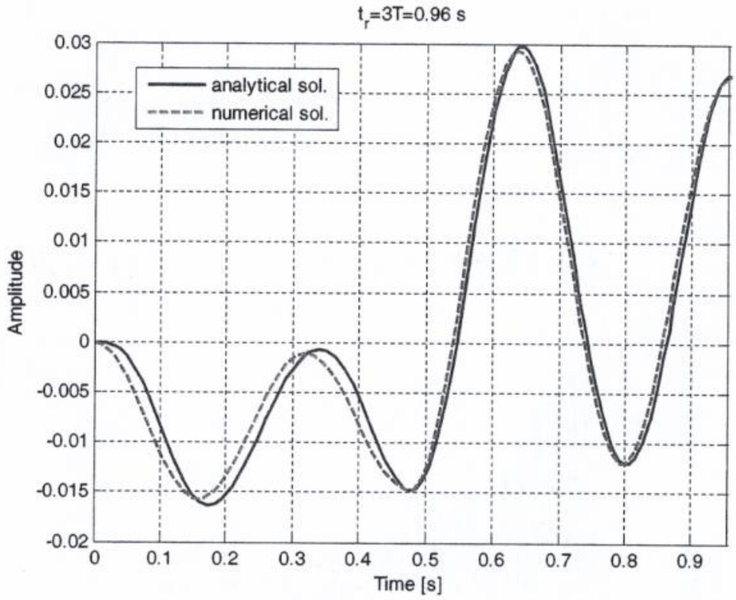

Figure 5. Rotational motion of the beam during rise time, t_r=3T.

Fig. 6 shows this change. Values are scaled with respect to the amplitude at m=1. At m=1, 3, and 5 maximum residual vibration amplitudes are making peaks. Between $1.5 < m < 2.5$ maximum vibration amplitude values are less than 3% of the amplitude for m=1. Between $3.5 < m < 4.5$ residual vibration amplitudes are less than 0.1% of the maximum vibration amplitude for m=1. These values are independent of the beam natural frequency that is why these results will not change for different beams.

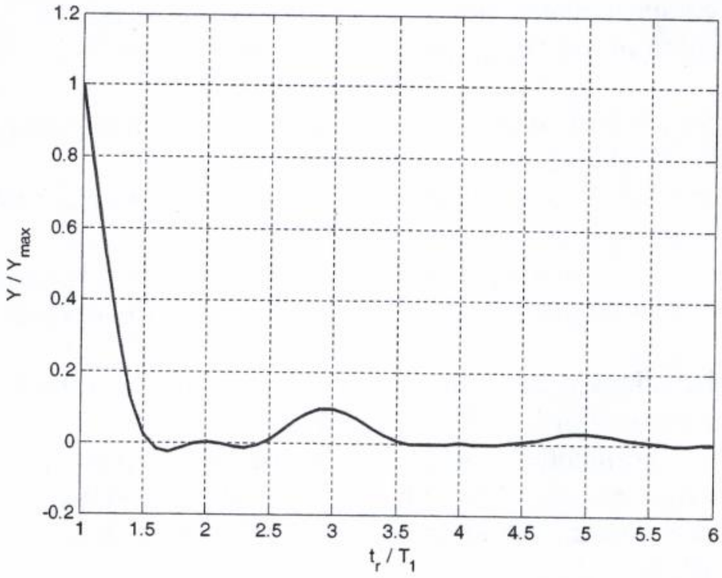

Figure 6. Residual vibration maximum amplitude spectrum.

4. CONCLUSION

In this study rotating flexible beam equations are derived. Assuming triangular velocity profile for the rotation, analytical and numerical solutions are obtained. Rotational acceleration of the beam is approximated by three term Fourier series expansion. Residual vibration maximum amplitude spectrum show that for the ratio of rise time to beam natural period values of 1, 3, 5, … maximum residual vibration amplitudes are maximized, for ratio values of 1.5 to 2.5, maximum residual vibration amplitudes are less than 3% of the value obtained for ratio=1. For ratio values of 3.5 to 4.5 maximum residual vibration amplitudes are less than 0.1% of the value obtained for ratio=1. This study shows that It is possible to minimize the residual vibration of the rotating beam by selecting proper rise time.

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